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ROYAL AIRCRAFT ESTABLISHMENT

FARNBOROUGH, HANTS

TECHNICAL NOTE No: G.W.174

THE LETHALITY OF THE BAR FRAGMENT TYPE OF WARHEAD:

AN ASSESSMENT WITH REFERENCE TO THE WARHEAD OF AN AIR-TO-AIR GUIDED WEAPON

by

G.T.J.PULLAN, M.A., F.E.DURANT, B.Sc. and R.G.KEATS, B.Sc.

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U.D.C. No. 623.451-519:623.4.081:623.566:623.562

Technical Note No. GW.174

February, 1952

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The Lethality of the Bar Fragment Type of Warhead: an Assessment with Reference to the Warhead of an Air-to-Air Guided Weapon

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R.A.E. Ref: GW/S/214/11/91

SUMMARY

This note contains a brief preliminary theoretical assessment of the lethality of a bar fragmenting warhead, with particular reference to the air-to-air attack of a Superfortress (B.29). Comparisons have been drawn between the effectiveness of the bar fragmenting warhead and the controlled fragmentation type. The accuracy of the guidance system has been considered in relation to the chances of hitting the target. A "constant distance" type of fuze has been assumed to be used for producing detonation, its performance being examined to determine the most effective burst distances and the degree of sensitivity required of such a fuze. Finally the factors likely to reduce the chance of hitting during the unguided flight of the bars towards the target after detonation have been analysed and an estimate made of their importance.

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1 Introduction

Trials carried out in the U.S.A. have shown that bars weighing approximately one pound, and having striking velocities of 2,000 ft/sec will cause considerable damage to the B-17 and FM-2 types of aircraft. The area of such aircraft vulnerable to bars is much greater than the corresponding area vulnerable to fragments while the bars are apparently more effective in producing immediate destruction of the aircraft. The lethality of a fragmenting warhead may be considerably reduced if the enemy should armour certain vital components of the target, notably the pilots and engines, but the possibility of reducing the area vulnerable to bars by any similar device would seem to be rather remote. For category A damage to unarmoured targets, however, the greater number of fragments which can be obtained for the same warhead weight may off-set the advantage of attacking the greater area vulnerable to bars. Very approximately, the product $N \times A$ is the important factor, where N is the number of bars or fragments, and A is the area of the target vulnerable to each. An estimate of this product for a 50 lb warhead controlled to give $\frac{1}{4}$ oz fragments would be 3,200 where $N = 1600$, $A = 2$ sq.ft; whereas for the bar type warhead of similar weight attacking a Superfortress, say, from astern the value of this product might be 3,500 where $N \doteqdot 50$, $A \doteqdot 70$ sq.ft. These figures suggest that against unarmoured targets the bar type warhead may have a performance comparable with that of the fragmenting type in effecting category A damage; hence it is desirable that more detailed studies of the efficiency of such a warhead should be made and such studies are described in the present note.

It has been assumed that the fuze is such that when the missile reaches some predetermined distance from the target the fuze actuates, detonating the warhead. The opportunity has been taken of finding whether any gain in lethality would result from the greater fuze actuation distances that might become available by use of such a "constant distance" fuze instead of the usual V.T. proximity fuze. The lethality of a bar fragment type of warhead is compared with that of a fragmenting type of similar weight and size attacking an aircraft target the pilots of which are armoured. The manner in which the lethality is influenced by inaccuracies in the guidance system by imperfections in the fuzing system, which cause the fuze to actuate before or after the missile reaches the predetermined distance from the target, has also been studied. Finally, the effect on lethality of the unguided flight of the bars towards the target, once the warhead has been detonated, is considered.

2 Vulnerability of Aircraft to Bars

Photographs received in this country of the results of U.S. trials of firing bars against various types of aircraft (FM2, B-17, SB2C) indicate the extent of the damage which may be inflicted on certain types of aircraft target by the bars, and it can be appreciated that such damage could probably cause immediate destruction of the aircraft[1].

For the purpose of the present study, the presented area, from astern, of the components of a Superfortress (B-29) vulnerable to bars has been estimated[1] to be 68 sq.ft of wings and 10 sq.ft of tail, with a possible further 35 sq.ft of fuel tanks which have not been included in the present calculations. These are the areas vulnerable to U.S. category A damage (destruction within 5 minutes); it is considered that 80% of these areas represents the area vulnerable to U.S. category K damage (immediate destruction). In the calculations the total area vulnerable to bars has been taken as 78 sq.ft.

The behaviour of the bars on hitting the target area - in particular, the manner in which they damage the aircraft - is not yet known in detail (see para.6), but is thought to depend on the mass, the velocity, and possibly the shape of the bars; however, no attempt has been made to incorporate these factors into the probabilities, which are expressed below purely as chances of hitting the target's vulnerable area or, briefly, as 'hit-chances'.

3 Description of the Mathematical Treatment

3.1 Representation of the Target and the Flight of the Bars

The target is considered to be a Superfortress (B-29) under attack by an air-to-air type of weapon. This weapon is designed for attacking targets from the rear, and the B-29 target as viewed from astern has been represented by an ellipse, the major axis of which corresponds to the wing span (about 140 ft) while the minor axis corresponds to the diameter of the fuselage (about 10 ft). The areas vulnerable to bar fragments are taken to be distributed uniformly over the ellipse. This simplifying assumption is considered to be valid if it is supposed that the fuze actuates the warhead at distances from the aircraft which are large compared with the overall length of the aircraft (about 100 ft). As has been mentioned the fuze actuates at some predetermined distance astern of the target, detonating an explosive charge which imparts to the bars a radial velocity, so that relative to the target they will appear to move forward within a cone, whose axis is the direction of the missile velocity, and whose plane semi-vertical angle is given by

$$\tan^{-1} \frac{\text{Maximum radial velocity of the bars}}{\text{Missile velocity, relative to the target}}$$

3.2 Derivation of Expressions for the Probability of Hitting the Target

The mathematical representation of the problem has been carried out for two slightly different assumptions. In the first case, when the bars have arrived at the target, they are assumed to be randomly distributed over the circular section of the cone of bar flight by the plane of the target area. This case is illustrated in Fig.1a, the probability expressions are derived in para.3.2a and are briefly discussed in section 5.4 in conjunction with the following case.

For the second case, this assumption has been modified so that the calculations correspond more closely to the behaviour of the bars as indicated by the trials results that are at present available[2]. These results suggest that the bars are thrown towards the target between the surfaces of two cones whose plane semi-vertical angles differ by a few degrees. Accordingly, in the second case, illustrated in Fig.1b it is assumed that on arrival at the target the bars are randomly distributed over the annular space between the circular sections of these cones by the plane of the target area. The probabilities based on this assumption are derived in para.3.2b.

In both cases the polar coordinates used have been taken in the plane of the elliptical target, with the centre of the ellipse as pole, and the major axis of the ellipse as the reference axis.

In the calculations below no account has been taken of the fragment velocity or the distance from the target at which the fuze is actuated, these quantities can be introduced at a later stage (paras. 4 and 5). Only the situation in the target area, once the bars have arrived, is considered in what follows; in this way is gained a wider applicability of the calculations and the results derived from them.

3.2a In Fig.1a, let (R,θ) be the coordinates of the point where the axis of the cone of bar flight meets the plane of the target area.

Let r be the radius of the circular section of the cone by the plane of the target area

Let a be the semi-major-axis of the ellipse) the ellipse being the
b be the semi-minor-axis of the ellipse) 'target area'

ΔA be the area common to the circle (containing the bars) and the ellipse

N be the number of bars

V be the area of the target vulnerable to bars.

Then, if the N bars are randomly distributed over the circle of radius r, the chance of any one bar hitting the target area $= \frac{\Delta A}{\pi r^2}$, and the chance of this bar hitting a vulnerable portion of the target area $= \frac{\Delta A}{\pi r^2} \cdot \frac{V}{\pi ab}$; then the chance that at least one bar will hit a vulnerable portion of the target area is given by

$$P_1\ (R,r,\theta) = 1 - \left[1 - \frac{\Delta A}{\pi r^2} \cdot \frac{V}{\pi ab}\right]^N \qquad (1)$$

ΔA is a function of R,r,θ, as can be seen from Fig.1a.

The average probability of a hit by at least one bar on a vulnerable portion of the target area when all values of the angle θ are considered to be equally likely, is given by

$$\frac{1}{2\pi}\int_0^{2\pi} P_1\ (R,r,\theta)d\theta \qquad (2)$$

If, in Fig.1a, the radius vector R is determined by vectorial addition of a mean bias L, and a scatter ρ about this mean; and α is the angle between the bias L and the axis of reference, while ϕ is the angle between the bias L and the scatter ρ, then the chance of hitting a vulnerable portion of the target area is given by

$$P_2\ (L,\alpha,\phi,r,\rho) = P_1\left[(L^2+\rho^2+2L\rho\cos\phi)^{\frac{1}{2}},\ r,\ \alpha \pm \sin^{-1}\left\{\frac{\rho\sin\phi}{(L^2+\rho^2+2L\rho\cos\phi)^{\frac{1}{2}}}\right\}\right]$$

When it is further supposed that the scatter, ρ , about the mean bias L, is gaussianly distributed with modulus h, then the average probability of hitting a vulnerable portion of the target area, for the case when the bias is L in direction α , is given by

$$P(L,\alpha,r,h) = \frac{h^2}{\pi} \int_0^{2\pi}\int_0^{\infty} P_2(L,\alpha,\phi,r,\rho)\, e^{-h^2\rho^2} \rho\, d\rho\, d\phi \qquad (3)$$

3.2b In Fig.1b, the quantities a,b,R,θ, are the same as in Fig.1a; in addition to these quantities, the following are defined:-

r_1, is the radius of the inner circular section

r_2, is the radius of the outer circular section

w, is the width of the ring between these circular sections; $w = r_2 - r_1$

ΔA_1, is the area common to this ring and the ellipse

Now when the N bars are assumed randomly distributed within this ring, the probability of one bar hitting a vulnerable portion of the target area becomes

$$P_3(R,\theta,r_1,r_2) = \frac{\Delta A_1}{\pi(r_2^2 - r_1^2)} \cdot \frac{V}{\pi ab} \qquad (4)$$

ΔA_1, is a function of R,θ,r_1, and r_2, as can be seen from Fig.1b. Further the probability that at least one bar will hit a vulnerable portion of the target area will be given by

$$P(R,\theta,r_1,r_2) = 1 - \left[1 - P_3(R,\theta,r_1,r_2)\right]^N \qquad (5)$$

The average probability of at least one hit on the vulnerable portion of the target area when all values of the angle θ are equally likely is

$$P(R,r_1,r_2) = \frac{1}{2\pi}\int_0^{2\pi} P(R,\theta,r_1,r_2)\, d\theta \qquad (6)$$

Geometrically the quantity R represents the point at which the trajectory of the missile, provided it had no lateral acceleration, would have passed through the plane of the target area. Hence the quantity R may be identified as the explicit miss distance.

If it is supposed that as a result of inaccuracies in the guidance system the missile trajectories are distributed normally about a central trajectory that will pass through the centre of the target area, then the miss distances must be distributed normally about a zero mean with modulus h. In these circumstances the average probability of hitting a vulnerable portion of the target area will be given by

$$P(r_1,r_2,\sigma) = \frac{h^2}{\pi}\int_0^{2\pi}\int_0^{\infty} P(R,\theta,r_1,r_2)\, e^{-h^2R^2} R\,dR\,d\theta, \qquad (7)$$

where $\sigma = \frac{1}{h}$ = R.M.S. radial miss distance.

In order to take account of the fluctuations in the fuze performance, the diagram of Fig.1c is required. In this figure, x_0 is the distance from the target where an ideal fuze would actuate the warhead; the angles α_1 and α_2 are the plane semi-vertical angles of the cones between which the bars fly towards the target; r_{10} and r_{20} are the radii of the circular sections of these cones in the plane of the target area. When the fuze actuates 'early', say at a distance x from the target, the angles of the cones remain unaltered (being governed by the initial velocity of the bars and the velocity of the missile relative to the target) but the radii of their circular sections in the plane of the target area are now r_1, and r_2.

By similar triangles, $\frac{r_1}{r_{10}} = \frac{x_1}{x_0} = \frac{r_2}{r_{20}}$

hence
$$\frac{\omega_1}{r_1} = \frac{\omega_o}{r_{10}} \qquad (8)$$

since $r_{20} = r_{10} + \omega_o$, $r_2 = r_1 + \omega_1$, and ω_1, ω_2 are the corresponding ring widths .

Using (8), $r_2 = r_1 + \omega_1$ becomes $r_2 = r_1 + \left(\frac{\omega_o}{r_{10}}\right) r_1$, then the probability of hitting a vulnerable portion of the target area when the radii of the bar annulus are assumed distributed normally with modulus h' about mean radii r_{10}, r_{20} is given by

$$P\,(r_{10}, \omega_o, \sigma, h') = \frac{h'}{\sqrt{\pi}} \int_0^\infty P\left[r_1, r_1 + r_1\left(\frac{\omega_o}{r_{10}}\right), \sigma\right] e^{-h'^2\,(r_1 - r_{10})^2} dr_1 \qquad (9)$$

This integral has been used to obtain the probabilities when variations in fuze actuation about some mean actuation distance are considered.

4 The Values of the Parameters

4.1 The warhead considered in the present work consists of 50 bars, each 15 inches long. The details of the arrangement of these bars round the charge have not been considered very closely, but it is thought that it may be possible to have them wrapped round a thin layer of charge, the central portion of the warhead being hollow. The amount of charge would depend on the lateral velocity it is desired to impart to the rods.

4.2 The relative velocity with which the rods are projected towards the target after detonation is compounded of the relative velocity of the missile with respect to the target, taken here as 2,000 ft/sec., and the lateral velocity imparted to the rods by the explosive charge. The target is assumed to be moving with a velocity of 500 ft/sec in a direction parallel to that of the missile. The early information available on the American damage trials which have been performed indicate that the resultant velocity of the rods on striking the target must be of the order of 2,000 ft/sec. Accordingly narrow cones of bar flight have been used in what follows, a lateral bar velocity of 500 ft/sec producing a cone of apparent total angle approximately 28°, while the resultant velocity of the bars on arrival at the target would be approximately 2060 ft/sec., (apart from the velocity loss due to air retardation).

Most of the results are derived for a lateral bar velocity of 500 ft/sec. However, some have been quoted for a lateral bar velocity of 1,000 ft/sec which would produce a cone of bar flight of apparent total angle about 54°. Larger lateral bar velocities would produce wider cones, but it has been found that they produce their effective lethalities (>10%) at shorter fuze actuation distances and longer miss distances than are contemplated here.

4.3 The explicit miss distance has been considered for various values from 0 to 100 ft from the centre of the target area. The radii of the circular sections have been chosen to ensure that for all the miss distances and fuze actuation distances considered, the annular ring will intersect the elliptical target area, and the width of the ring has been kept narrow in view of the indications from trials that this in fact is the case, but a range of widths about the mean annular radius, $\pm 2\frac{1}{2}$, ± 5, ± 10, ± 40 ft, has been chosen to examine the effect of ring width on the hit chance.

The values of L, ρ and α used to evaluate expression (3) have been suggested by such preliminary information as is available - this indicates that the R.M.S. value of the scatter ρ is likely to be approximately 35 ft, and it is hoped that the bias L will be considerably less than 15 ft. However, the range of values of both L and ρ investigated is 0 to 80 ft and because of target symmetry the expression has been evaluated only for $0 \leq \alpha \leq \pi/2$.

5 Results

5.1 Definition of Limiting Case

The expressions (3), (6), (7) and (9) of section 3.2 have been evaluated for the chosen ranges of parameters, and the results expressed in the form of graphs in Figs.(3) to (8).

The number of graphs required because of the introduction of the ring width parameter, ω, has been reduced and, in some cases the computing simplified, by studying a limiting case, in which it is supposed that on arrival at the target area the bars are randomly distributed around the circumference of a circle obtained when $r_1 \to r_2$ and $\omega \to 0$ in Fig.1b. Some calculations have been made (para.5.5 and Table I) which justify this use of a 'zero ring width' to summarize results.

These results have been expressed in terms of 'hit chances' rather than 'kill chances', since the precise manner in which the bars damage the target is, as was noted in section 2, still uncertain.

5.2 Comparison Between Bar Fragmenting and Controlled Fragmentation Warhead

When a bar fragment type of warhead is compared with a controlled fragmentation type of the same weight and similar dimensions, both assumed to be attacking a target with its crew armoured, it can be seen, from Fig.2, that for explicit miss distances less than 55 feet the controlled fragmentation warhead produces higher probabilities of destruction than the bar fragment type produces hit chances. At explicit miss distances greater than 55 feet the probability of destruction by the small fragments falls off steadily, but at such miss distances the bars are capable of producing higher chances of hitting. The quality of damage produced needs to be considered in interpreting these trends, and it would appear that the $\frac{1}{4}$ oz cube-shaped fragments, which are the most effective small fragments, would be useful generally in producing damage of the U.S. "A"

category, while the bars would tend to produce the U.S. "K" category - this most important property of the bars has not yet been completely established by the U.S. field trials results at present available. It may be noted in Fig.2 that $\frac{1}{8}$ oz cubical fragments are the least effective while the $\frac{1}{2}$ oz fragments are intermediate between them and the $\frac{1}{4}$ oz size, which is the size favoured for the Blue Sky controlled fragmentation warhead. Further features of Fig.2 are the dependence of the hit chance of the bars on both fuze burst range and explicit miss distance, and the suggestion that the shortest fuze burst distance considered produces the highest absolute hit chance (curve No.1), which occurs at an explicit miss distance of 20 feet, but falls off very rapidly for explicit miss distances greater or less than this.

5.3 Probability Contours for the Chance of Hitting

The chance of obtaining a hit on a vulnerable portion of the target can be calculated, for any position at which the warhead is detonated, by means of expression (6) of section 3. Using these results, contours of equal hit chances can be drawn for burst points to the rear of the target aircraft. These contours, which represent the hit chances averaged with respect to the angle of inclination to the horizontal of the explicit miss distance (see Fig.1a) are drawn in relation to the target in Fig.3. These curves show where the missile must be detonated in order to have a given chance of hitting the target. When the fuze is actuated at distances between 100 ft and 200 ft from the target on trajectories whose explicit miss distances are between 20 ft and 50 ft the hit chance varies between 20% and 40%. These curves show also that the most profitable attacks are likely to occur when the fuze actuates close to the target, at distances possibly less than 100 ft.

5.4 Effect of Guidance Accuracy

Once the guidance system of the missile is taken into account the miss distances must be assumed to be gaussianly distributed about a zero mean, and expression (7) of section (3) has to be used. The graphs of Fig.4 were so obtained, to show the variation of the hit chances with R.M.S. miss distance for various fuze actuation distances. The indications are that the shortest fuze actuation distances considered are the best for most R.M.S. miss distances. This is in general confirmed by the curves of Fig.7 in which variations about the pre-set fuze actuation distance are included, but it has to be modified when the R.M.S. miss distance exceeds 60 feet, for then in general the best fuzing distance is about twice the R.M.S. miss distance.

The assumption that the R.M.S. miss distances are gaussianly distributed about a zero mean is not necessarily valid. It has been suggested that a bias is introduced by the guidance system, as a result of which the trajectories along which the missile may be directed must be supposed distributed about a trajectory which does not pass through the centre of the target area. So, for any given trajectory the miss distance is taken to be the vector sum of a vector L, representing the bias, and a vector ρ, called the scatter. It is then supposed that the scatter is gaussianly distributed about this bias. The calculations accounting for this, have been derived in section 3.2a, the quantities involved are illustrated in Fig.1(a). The graphs of Figs.5a to 5d - the results of evaluating expression (3) - show the variation of the hit chance with fuze actuation distance for various values of the bias and several combinations of the scatter and the angle α. Present ideas are that the bias will not exceed 15 ft and the scatter about this will not be more than 35 ft; so for a given fuze actuation distance of say 120 ft the hit chance would be of the order of 25%. The curves indicate, moreover, that fuze distances of not greater than 120 ft will tend to be the most effective for

the values of bias and scatter likely to occur. It should also be noted that these curves indicate that there appears to be an advantage in ensuring that the angle α is nearly zero, or, in other words, accuracy perpendicular to the direction of the wings of the target is more important than accuracy in the direction of the wings.

5.5 Fuze System Performance

If it is assumed that the performance of the fuze system is such as to actuate the warhead at distances gaussianly distributed about the pre-set distance, it has been found that such fluctuations in fuze performance have not a marked influence on the hit chance. In Fig.6, which shows the variation of hit chance with R.M.S. miss distance for certain values of the S.D. about various actuation distances, it is clear that considerable latitude is permissible in the fuze performance before deterioration in the hit chance occurs. This is true for both lateral bar velocities considered, 500 ft/sec and 1,000 ft/sec., though for the higher velocity there is less tolerance available.

The curves of Fig.7 show that as the S.D. about the mean actuation distance increases, the variation of hit chance becomes less dependent on the fuze actuation distance. It has already been noted in previous paragraphs, and it is confirmed in these curves that the shortest fuze actuation distance considered is the most useful. The dependence on fuze actuation distance of the maximum hit chance for a given explicit miss distance is shown in Fig.8a, but when guidance accuracy is taken into account, as is done in Fig.8b, the maximum hit chance depends much less on the actuation distance; indeed the curves of Fig.8b indicate that the optimum value of the distance for detonating the warhead is approximately 80 feet.

5.6 Unguided Flight of Bars Towards the Target After Detonation

After the fuze has been actuated it may be presumed that the unguided bars will be less accurate than would have been the guided missile itself, and it is of interest to investigate the loss of accuracy due to the unguided flight of the bars. If it is permissible to assume that the motion of the missile is governed by a linear transfer function, then we may consider the scatter and bias independently. Assuming first of all that all deviations of the missile from the line of sight from the fighter to the target are due to scatter, then taking this line of sight as the z-axis and the axes of the target as the x- and y-axes, the motion of the centre of gravity of the bars is defined by the values $x_A, \dot{x}_A; y_A, \dot{y}_A; z_A, \dot{z}_A$ describing the displacement and velocity of the bars at the time of fuze actuation. Then, if t is the time of flight of the bars

$$\rho^2 = (x_A + t\dot{x}_A)^2 + (y_A + t\dot{y}_A)^2$$

which is $\rho^2 = x_A^{\ 2} + 2tx_A\dot{x}_A + t^2\dot{x}_A^{\ 2} + y_A^{\ 2} + 2ty_A\dot{y}_A + t^2\dot{y}_A^{\ 2}$

Therefore, since x_A and $\dot{x}_A$ are uncorrelated,

$$\sigma_\rho^{\ 2} = \sigma_s^{\ 2} + 2t^2\,\sigma_\ell^{\ 2} \tag{10}$$

where $\sigma_s^{\ 2}$ = variance of the scatter of the missile,

$\sigma_\ell^{\ 2}$ = variance of the missile velocity in the x- or y-direction, assuming these to be equal.

Similarly, there may be assumed a condition in which there is no scatter, but the centre of gravity of the bars is displaced from the line of sight at the time of fuze actuation. The bars will then retain the velocity of the missile, but they will have no acceleration, and so, after time t will be further biassed by a further distance equal to

$$\tfrac{1}{2} gt^2 \qquad (11),$$

where g is the lateral acceleration of the missile due to target manoeuvre. Hence, due to the unguided motion of the bars there will be an increased bias equal to $\frac{1}{2} gt^2$, and an increase in the scatter variance equal to $2t^2 \sigma_\ell^2$. Both these quantities will be small if t is small, hence there is an incentive to reduce the time of flight of the bars as much as possible. However, as the time of flight largely determines the dispersion of the bars, it cannot be reduced to zero.

Let v = maximum lateral velocity that can be given to the bars

u = relative velocity of the missile and target

s = distance of the missile from the target when the fuze is actuated,

then $r = vt$, $t = s/u$ and $s = \frac{ur}{v}$.

Reasonable values of these parameters are u = 1,500 ft/sec., r = 40 feet, v = 500 ft/sec., s = 120 feet and t = 0.08 sec. Substituting this value of t in (10) and (11) above it is clear that, unless σ_ℓ^2 is unexpectedly large no deterioration in performance may be expected due to the unguided flight of the bars. No values for σ_ℓ^2 are as yet available.

6 Postscript

Since the above was written further information, from design studies on the warhead of possible air-to-air missiles has become available, which suggests that the use of fifty as the number of bars which could be used in the warhead was optimistic; the number which, it would appear, is more realistic, is nearer twenty-five - always supposing, of course, that the length is maintained at 15 inches. The effect of this on the above work is to reduce the absolute values of the hit chances obtained (section 3, expressions (1) and (5)); the general behaviour of the hit chances in the various cases considered is, however, unaffected.

Some trials, carried out in this country, using bars against spar booms have suggested that the lethality of bars may have been overestimated by the early workers. While no account has been taken in this work of damage done by the bars after striking the target (section 2), it may be necessary to consider using bars not at a velocity of approximately 2,000 ft/sec (section 4.2) but at say 5,000 ft/sec. This would again raise the question as to whether an orthodox VT fuze, or a 'constant distance' type, considered here, would be the more desirable - quite possibly the former, particularly as this would avoid the need for research and development towards the design of a new fuzing technique.

7 Further Work

The present note has studied bar fragments with reference to an attack on an aircraft by a missile flying on a straight course parallel to the target and astern of it; other directions of attack, and other target aspects may require study, to extend the appreciation of the lethality of the bar fragment type of warhead.

Trials are planned from which it is hoped to obtain information on the behaviour of the bars when they strike the target, and a determination of the threshold velocity below which the bars cease to damage the target.

Means of increasing the hit chance may have to be sought, one of these being to increase the number of bars by reducing their size so that more bars are available in the same weight of warhead. The smallest size of bar that is effective must be determined from trials.

8 Conclusions

1 The available evidence suggests that bar fragments are more effective than orthodox fragmentation for inflicting U.S. 'K' category of damage (immediate destruction), but are likely to be less useful for producing the U.S. 'A' category (destruction within 5 minutes). The $\frac{1}{4}$ oz cubical fragments, the size adopted for use in the controlled fragmentation type warhead, seem likely to be more generally useful for producing 'A' type damage.

2 The best actuation distance for initiating a bar fragment warhead employing a 'constant distance' type of fuze, has been shown to be short, possibly less than 100 ft from the target, even for bars leaving the warhead at a low (500 ft/sec) lateral velocity and for a missile to target relative velocity of about 2000 ft/sec.

3 Considerable latitude in the sensitivity of the performance of the 'constant distance' type of fuze is permissible; imperfections in performance have been shown to be unlikely to have as much effect on the hit chance as the guidance accuracy.

4 Once the warhead has been detonated, there is very little deterioration of the hit chance to be expected due to the unguided flight of the bars towards the target.

5 The above calculations taken in conjuction with the few results available from trials in this country suggest that the most effective manner of employing bar fragments would be to design a warhead producing high speed bars, and embodying a VT fuze for the initiation; such a proposal merits further experimental and theoretical investigation.

9 Acknowledgement

The authors desire to acknowledge the assistance of Mr. F.I. Reynolds and members of the Computing Section of Assessment Division in carrying out a large part of the numerical work organized for the above work.

LIST OF REFERENCES

No.	Author	Title, etc
1	W.H. Stephens, F.W.R. Bird, W.R.B. Hynd	'Aircraft Vulnerability and Weapon Lethality Developments in the U.S.A.' GW Report 7, Mech. Eng. No.15
2		"Report of British Mission to U.S.A. on Aircraft Vulnerability and A.A. Lethality" A.A. Lethality Paper No.32

Attached: Table I
Drawings GW/P/3240 to 3250

Advance Distribution

M.O.S. Headquarters		R.A.E.	
Chief Scientist		Director	
CGWL		DD(E)	
CS(A)		DD(A)	
PDSR(A)		RPD	2
DGTD(A)		Aero	
DGWRD	Action copy + 10	Arm	2
DMARD		ME Dept	
D Arm RD		" Mr. F.W. Bird	
GWAB (Dr. R.C. Knight)		GWTW	3
TPA3/TIB	175	Library	

TABLE I

Comparison of the chance of hitting the target area when the bars are supposed randomly distributed in annuli of various widths about the radius of the circular section (of the cone of bar projection) by the plane of the target area

σ \ r_m	20 ± 0	20 ± 5	20 ± 10	40 ± 0	40 ± 20	60 ± 0	60 ± 20	60 ± 40	80 ± 0	80 ± 20	80 ± 40	100 ± 0	100 ± 20
20	0.435	0.425	0.420	0.226	0.220	0.092	0.090	0.096	0.021	0.027	0.038	0.001	0.001
40	0.275	0.280	0.282	0.205	0.202	0.111	0.109	0.100	0.046	0.048	0.053	0.015	0.016
60	0.171	0.175	0.180	0.155	0.155	0.110	0.108	0.096	0.065	0.065	0.065	0.034	0.035
80	0.112	0.125	0.125	0.110	0.114	0.090	0.090	0.084	0.067	0.066	0.062	0.045	0.045

r_m = Mean radius of the circular section by the target area

σ = R.M.S. miss distance (feet)

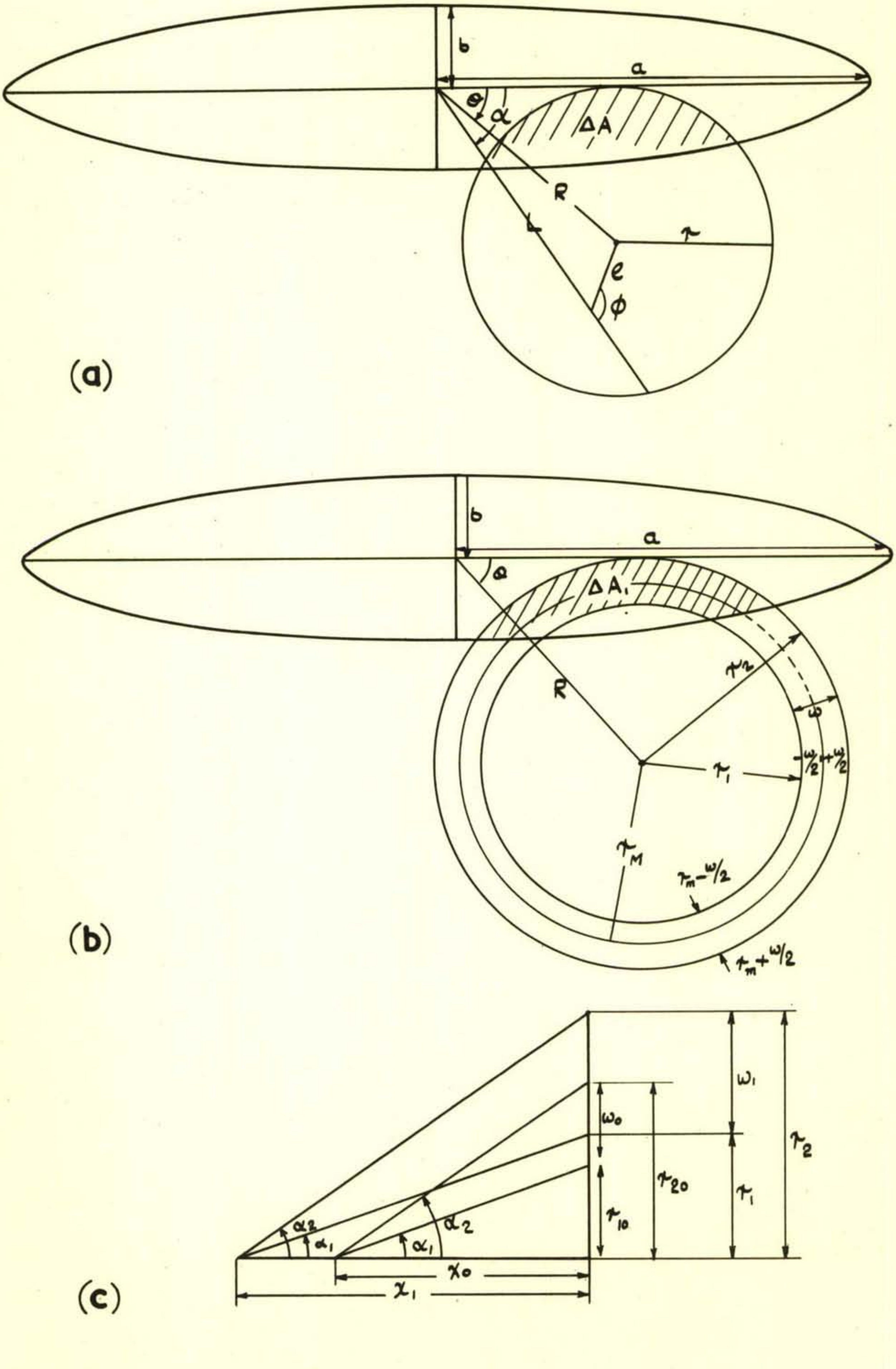


FIG 1.a.b.c. FIGURES REPRESENTING THE GEOMETRY INVOLVED IN THE CALCULATIONS.

FIG. 2.

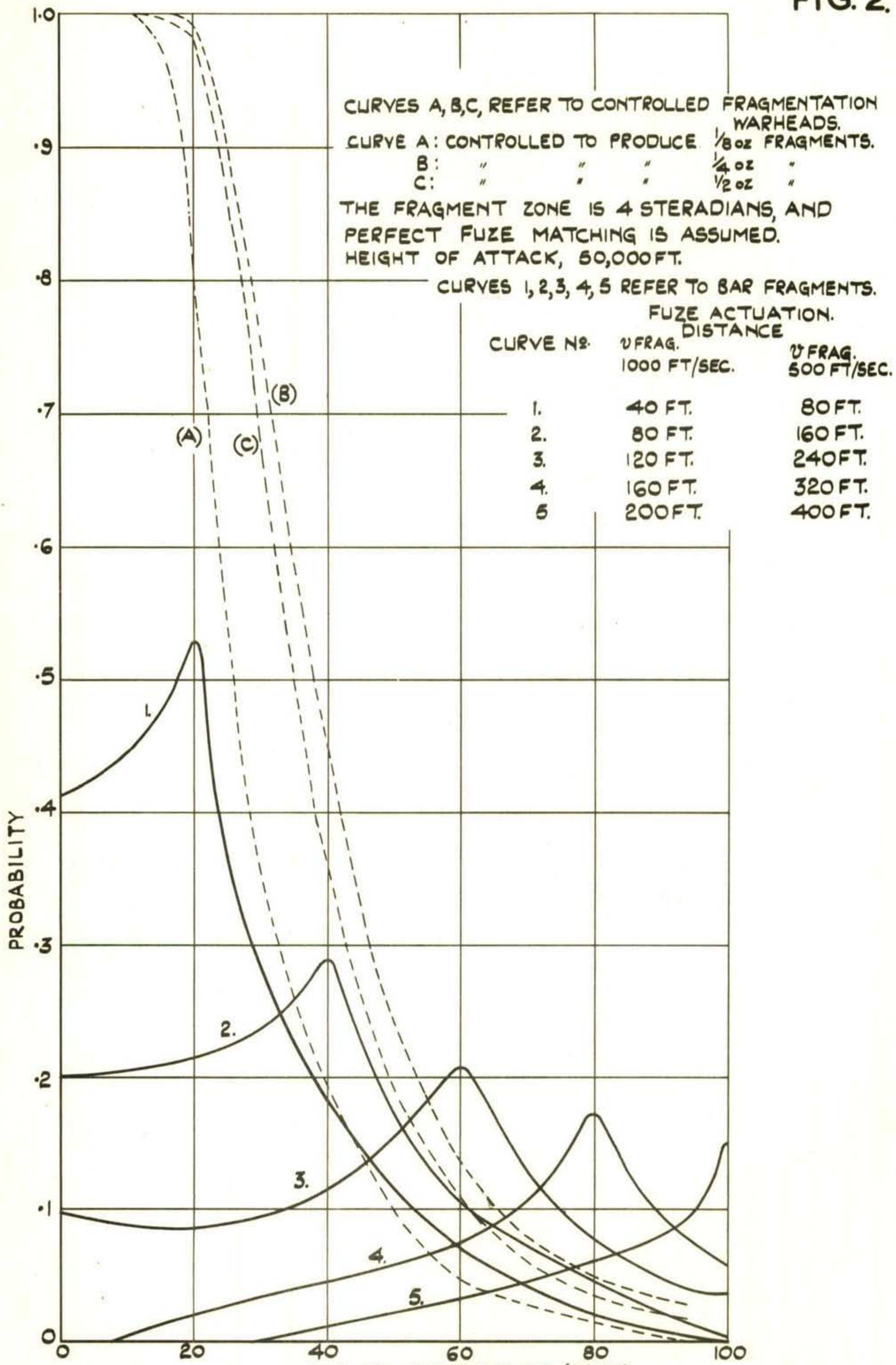
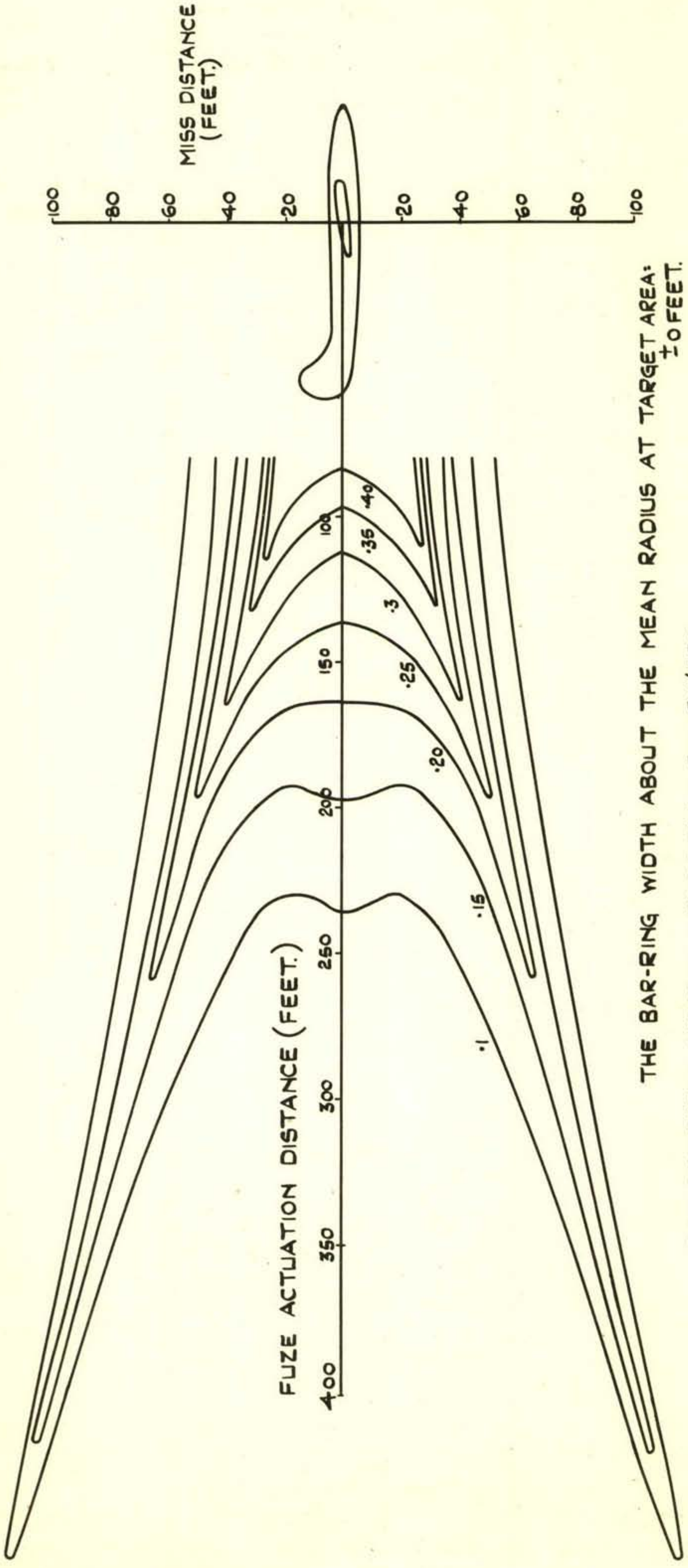


FIG. 2. COMPARISON BETWEEN THE VARIATION OF HIT CHANCE WITH EXPLICIT MISS DISTANCE OF BAR FRAGMENTS AND THE VARIATION OF KILL CHANCE WITH EXPLICIT MISS DISTANCE OF FRAGMENTS FROM A CONTROLLED FRAGMENTATION WARHEAD OF SIMILAR DIMENSIONS AND TOTAL WEIGHT, WHEN ATTACKING A FOUR ENGINED BOMBER THE PILOTS OF WHICH ARE ARMOURED.



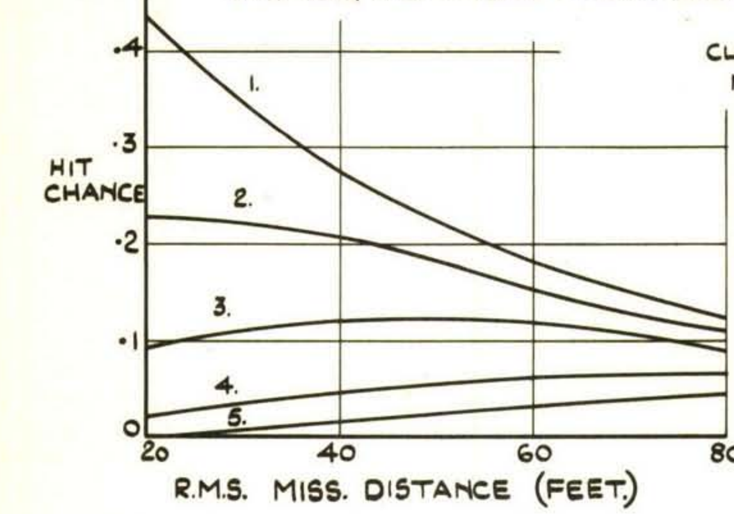
THE BAR-RING WIDTH ABOUT THE MEAN RADIUS AT TARGET AREA: ±0 FEET.

BAR FRAGMENT LATERAL VELOCITY = 500 FT/SEC.
MISSILE VELOCITY = 2,500 FT/SEC.
TARGET VELOCITY = 500 FT/SEC.

FIG. 3. EQUAL HIT CHANCE CONTOURS.

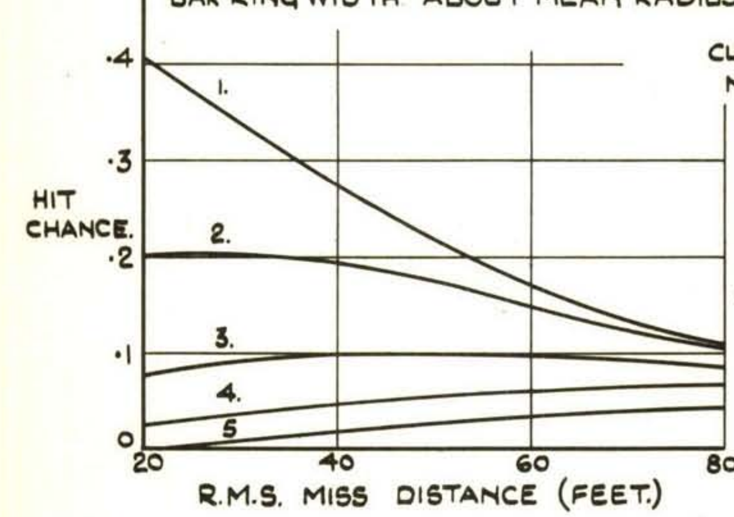
FIG.4.

BAR RING WIDTH ABOUT MEAN RADIUS AT TARGET AREA = 0 FEET.



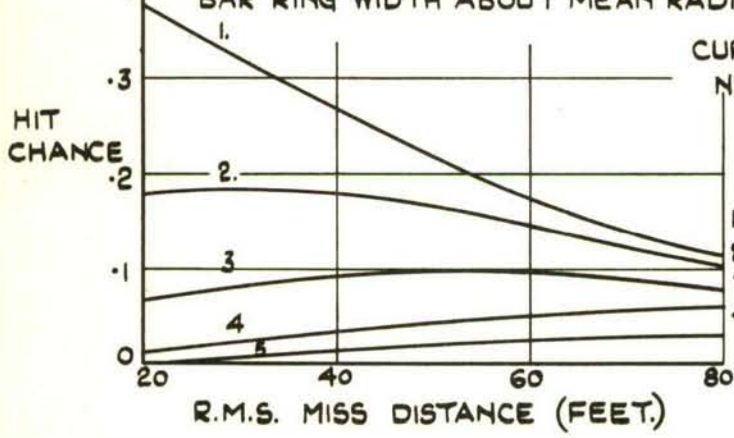
CURVE Nº.	MEAN RADIUS OF CIRCULAR SECTION BY PLANE OF TARGET AREA. (FEET.)	FUZE ACTUATION DISTANCE (FEET) FRAG. VEL. 500 FT/SEC.	FRAG. VEL. 1000 FT/SEC.
1.	20.	80	40
2.	40.	160	80
3.	60.	240	120
4.	80.	320	160
5.	100.	400	200

BAR RING WIDTH ABOUT MEAN RADIUS AT TARGET AREA = ±2·5 FEET.



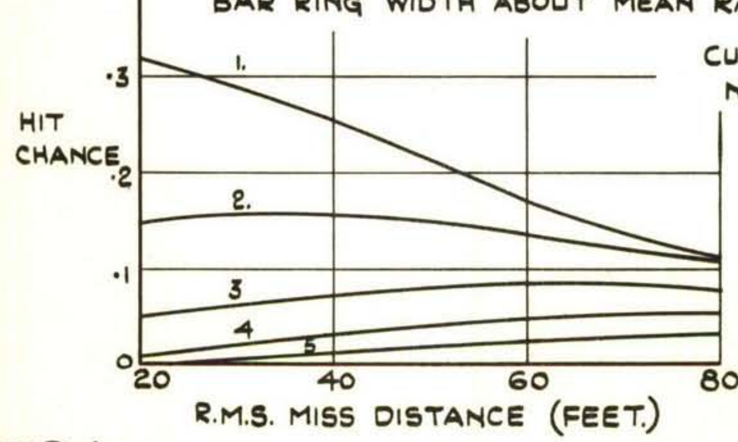
CURVE Nº.	MEAN RADIUS OF CIRCULAR SECTION BY PLANE OF TARGET AREA.	FUZE ACTUATION DISTANCE (FEET.) vFRAG = 500 FT/SEC.	vFRAG = 1000 FT/SEC.
1.	22·5	90	45
2.	42·5	170	85
3.	62·5	250	125
4.	82·5	330	165
5.	102·5	410	205

BAR RING WIDTH ABOUT MEAN RADIUS AT TARGET AREA = ±5 FEET.



CURVE Nº.	MEAN RADIUS OF CIRCULAR SECTION BY PLANE OF TARGET AREA.	FUZE ACTUATION DISTANCE (FEET.) vFRAG. = 500 FT/SEC.	vFRAG. = 1000 FT/SEC.
1.	25.	100.	50.
2.	45.	180.	90.
3.	65.	260.	130
4.	85.	340.	170.
5.	105.	420.	210.

BAR RING WIDTH ABOUT MEAN RADIUS AT TARGET AREA = ±10 FEET.



CURVE Nº.	MEAN RADIUS OF CIRCULAR SECTION BY PLANE OF TARGET AREA.	FUZE ACTUATION DISTANCE (FEET.) vFRAG. = 500 FT/SEC.	vFRAG. = 1000 FT/SEC.
1.	30.	120.	60.
2.	50.	200.	100.
3.	70.	280.	140.
4.	90.	360.	180.
5.	110.	440.	220.

FIG.4. VARIATION OF THE HIT CHANCE WITH THE R.M.S. MISS DISTANCE, ASSUMING A GAUSSIAN DISTRIBUTION OF MISS DISTANCES ABOUT ZERO MEAN. VARIOUS FUZE ACTUATION DISTANCES ARE CONSIDERED, AND BAR-RING WIDTHS.

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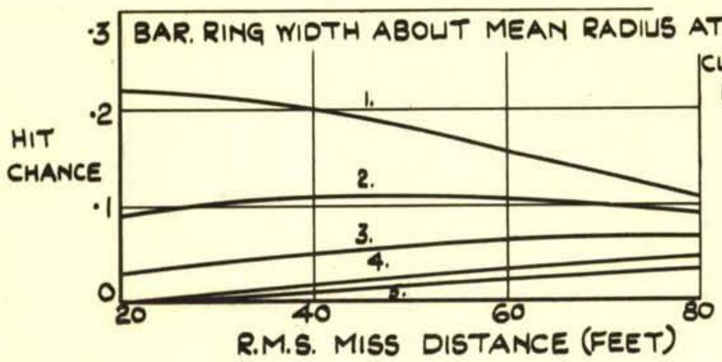
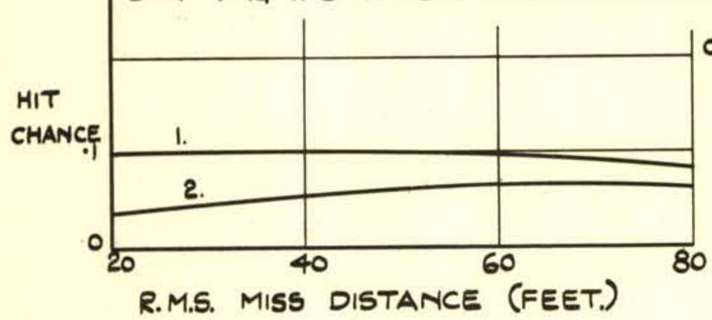
FIG. 4. CONTD.

BAR. RING WIDTH ABOUT MEAN RADIUS AT TARGET AREA = ± 20 FEET.



CURVE Nº	MEAN RADIUS OF CIRCULAR SECTION BY PLANE OF TARGET AREA.	FUZE ACTUATION DISTANCE $v_{FRAG.}$ = 500 FT/SEC.	FUZE ACTUATION DISTANCE $v_{FRAG.}$ = 1000 FT/SEC.
1.	40.	160.	80.
2.	60.	240.	120.
3	80.	320.	160.
4	100.	400.	200.
5.	120.	480.	240.

BAR RING WIDTH ABOUT MEAN RADIUS AT TARGET AREA. = ± 40 FEET.



CURVE Nº	MEAN RADIUS OF CIRCULAR SECTION BY PLANE OF TARGET AREA.	FUZE ACTUATION DISTANCE $v_{FRAG.}$ = 500 FT/SEC	FUZE ACTUATION DISTANCE $v_{FRAG.}$ = 1000 FT/SEC.
1	60	240	120
2	80	320	160

FIG. 4. CONTD. VARIATION OF THE HIT CHANCE WITH THE R.M.S. MISS DISTANCE, ASSUMING A GAUSSIAN DISTRIBUTION OF MISS DISTANCES ABOUT ZERO MEAN. VARIOUS FUZE ACTUATION DISTANCES ARE CONSIDERED, AND BAR-RING WIDTHS.

FIG. 5 a.

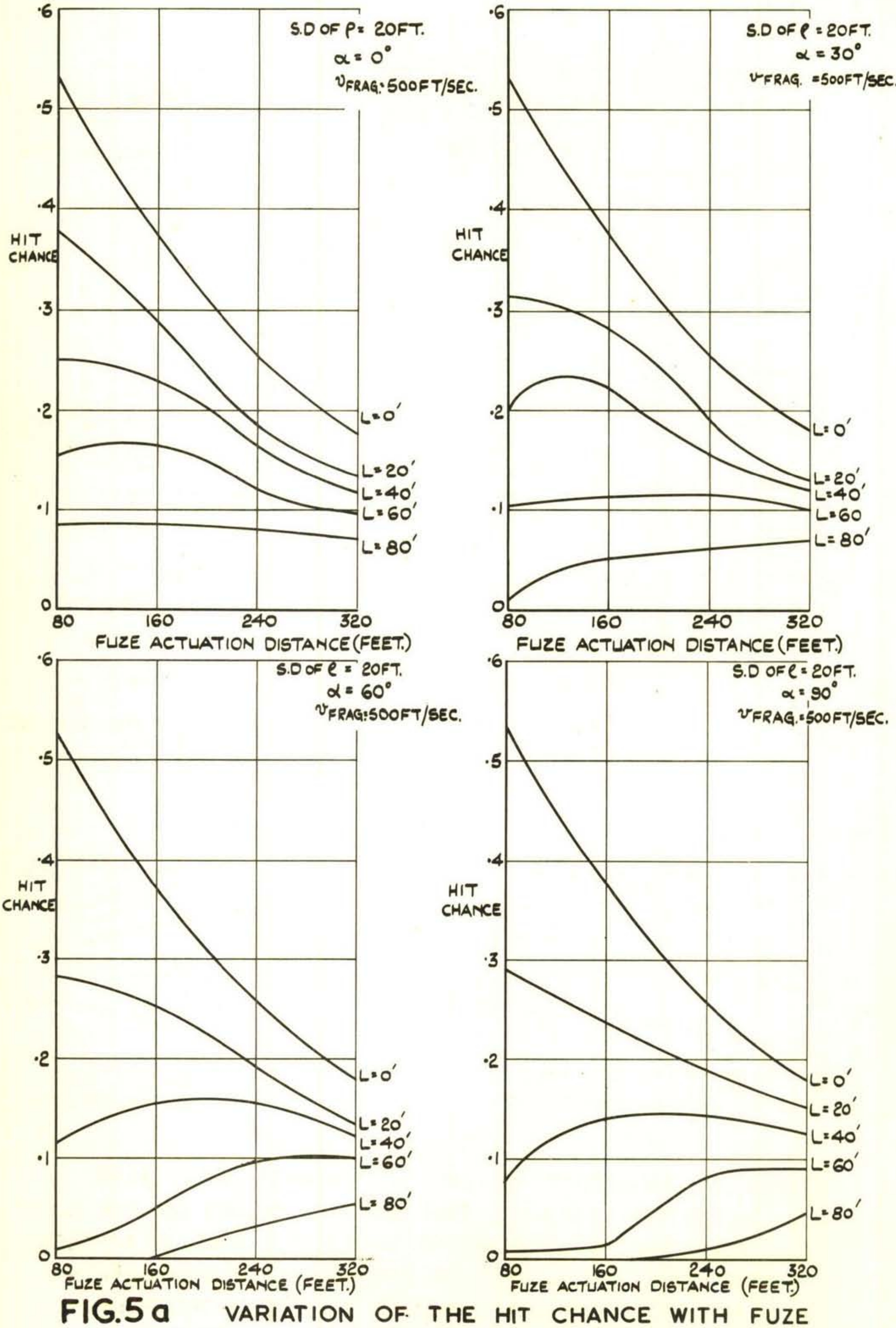


FIG. 5 a VARIATION OF THE HIT CHANCE WITH FUZE ACTUATION DISTANCE, FOR VARIOUS VALUES OF THE BIAS WHEN THE R.M.S. SCATTER ABOUT THE BIAS IS 20 FEET, AND FOR FOUR VALUES OF THE ANGLE OF INCLINATION (α) OF THE BIAS VECTOR TO THE HORIZONTAL.

FIG. 5 b.

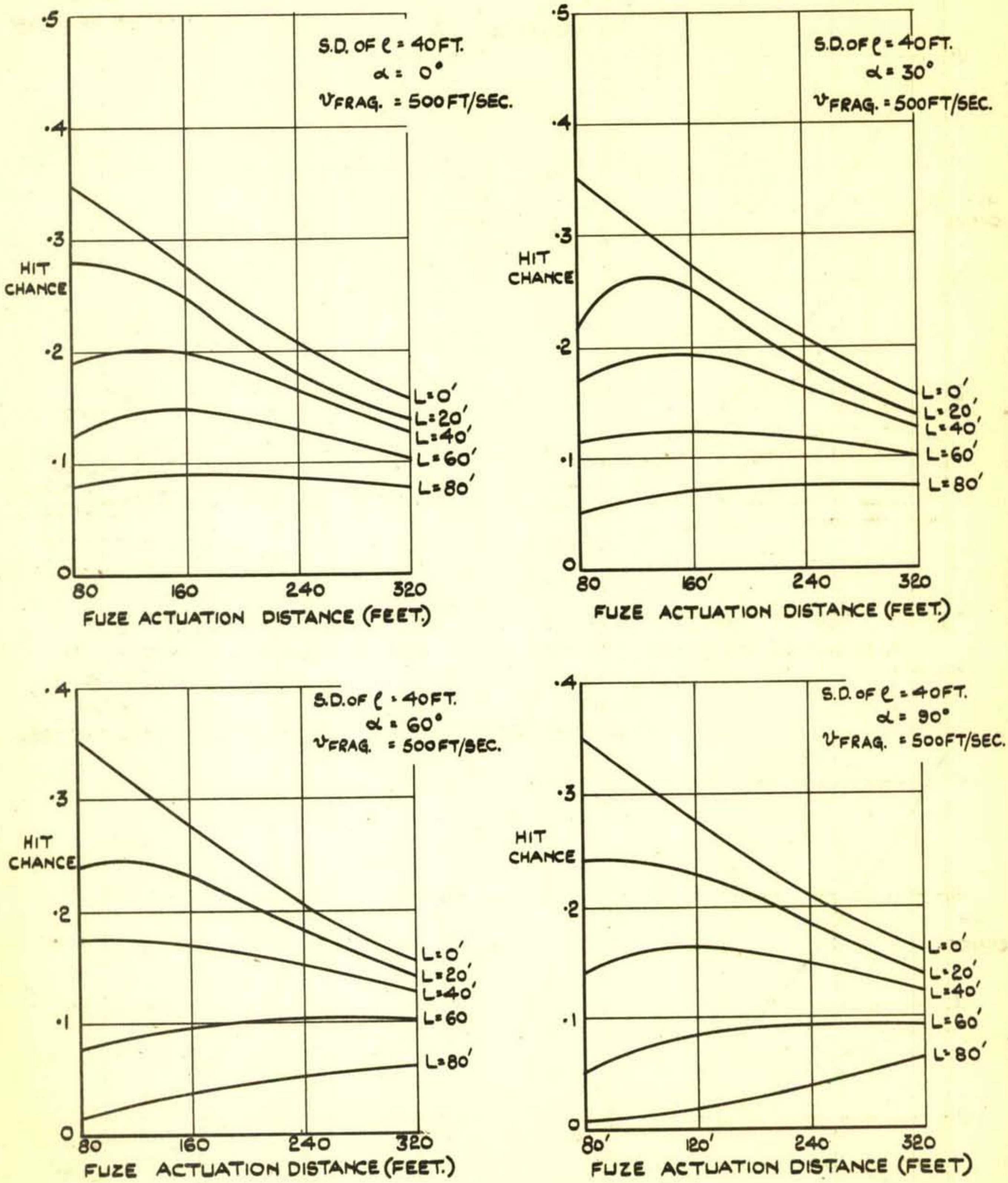


FIG. 5 b. VARIATION OF THE HIT CHANCE WITH FUZE ACTUATION DISTANCE, FOR VARIOUS VALUES OF THE BIAS WHEN THE R.M.S. SCATTER ABOUT THE BIAS IS 40 FEET, AND FOR FOUR VALUES OF THE ANGLE OF INCLINATION (α) OF THE BIAS VECTOR TO THE HORIZONTAL.

FIG. 5.(c & d)

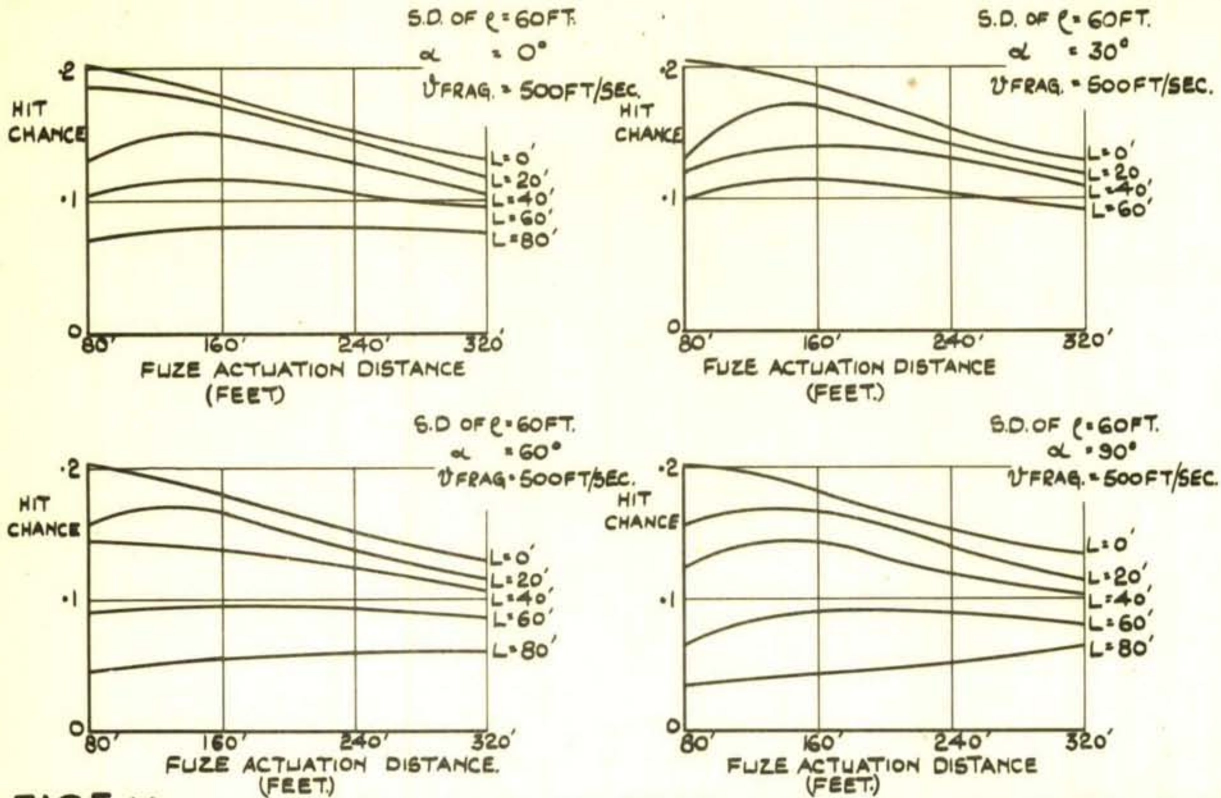


FIG.5.(c) VARIATION OF THE HIT CHANCE WITH FUZE ACTUATION DISTANCE FOR VARIOUS VALUES OF THE BIAS WHEN THE R.M.S. SCATTER ABOUT THE BIAS IS 60FT. & FOR FOUR VALUES OF THE ANGLE OF INCLINATION (α) OF THE BIAS VECTOR TO THE HORIZONTAL.

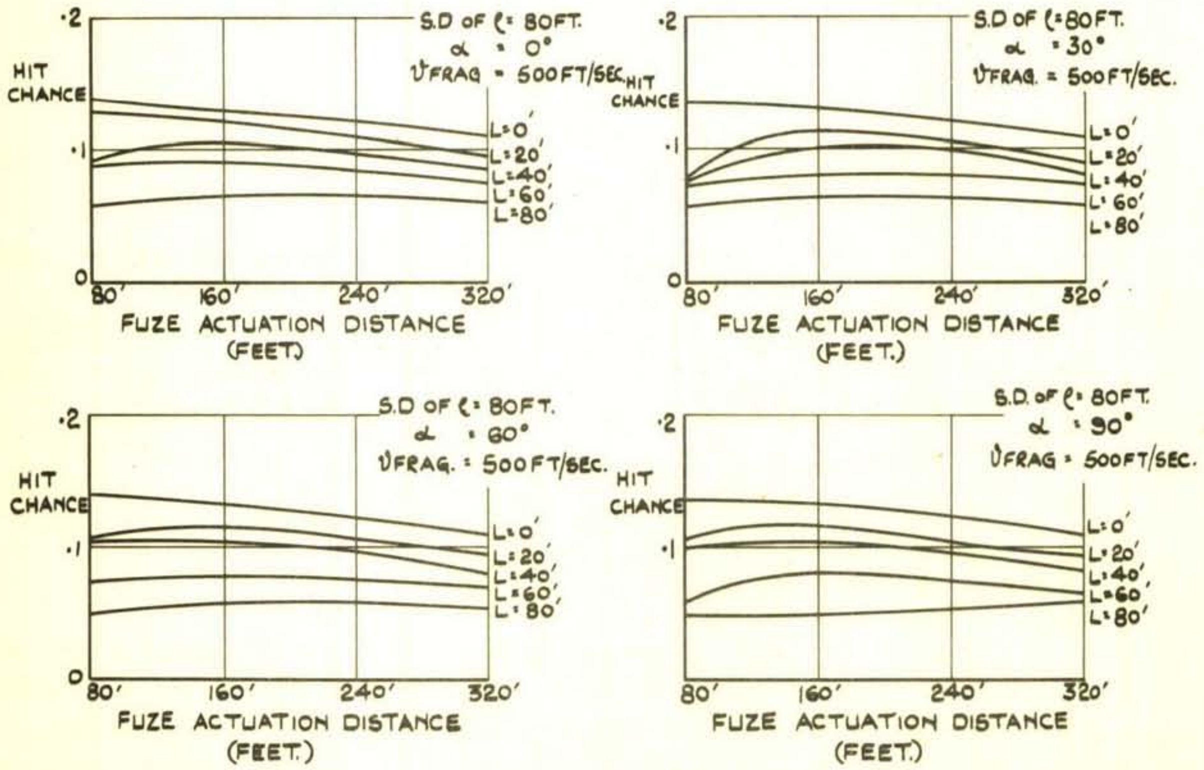
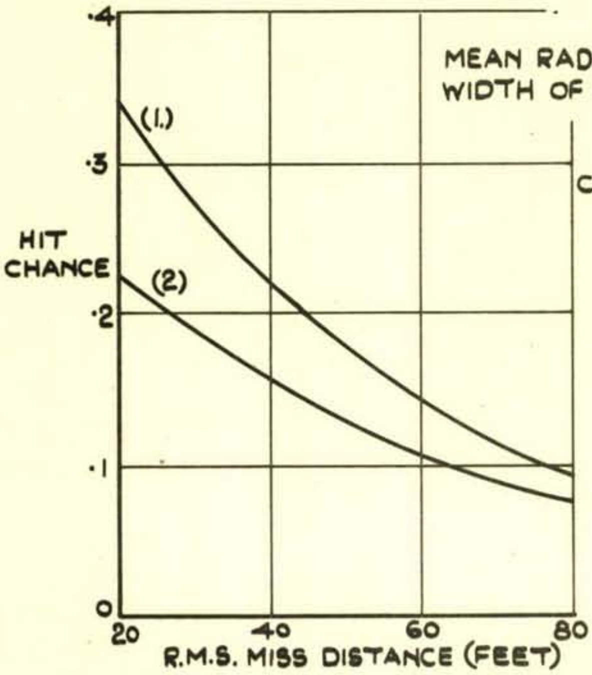


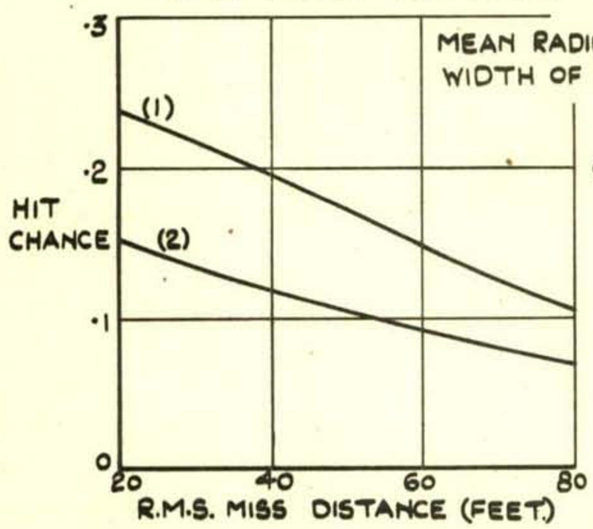
FIG.5.(d) VARIATION OF THE HIT CHANCE WITH FUZE ACTUATION DISTANCE, FOR VARIOUS VALUES OF THE BIAS WHEN THE R.M.S. SCATTER ABOUT THE BIAS IS 80FT., AND FOR FOUR VALUES OF THE ANGLE OF INCLINATION (α) OF THE BIAS VECTOR TO THE HORIZONTAL.

FIG. 6.



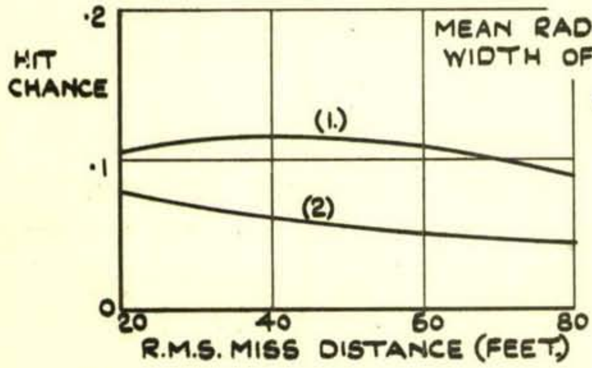
MEAN RADIUS OF BAR-RING IN PLANE OF TARGET AREA = 20FT.
WIDTH OF RING ABOUT MEAN RADIUS = ±0FT.

CURVE No.	FRAG. VEL. 500 FT/SEC. MEAN FUZE ACTUATION DISTANCE.	FRAG. VEL. 500 FT/SEC. STANDARD DEVIATION APPROX.	FRAG. VEL. 1000 FT/SEC. MEAN FUZE ACTUATION DISTANCE.	FRAG. VEL. 1000 FT/SEC. STANDARD DEVIATION APPROX.
1.	80.′	57′	40′	28′
2.	80′	114′	40′	57′



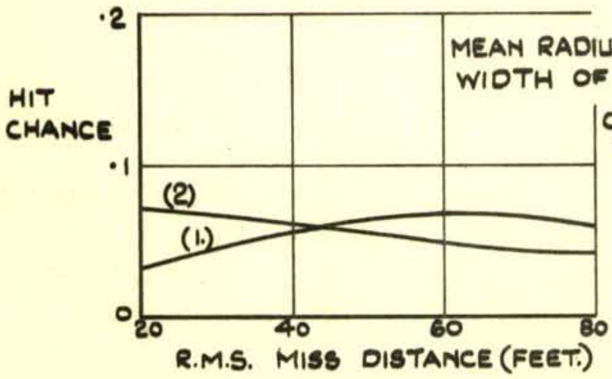
MEAN RADIUS OF BAR RING IN PLANE OF TARGET AREA = 40FT.
WIDTH OF RING ABOUT MEAN RADIUS = ±0FT.

CURVE No	FRAG. VEL. 500 FT/SEC. MEAN FUZE ACTUATION DISTANCE	FRAG. VEL. 500 FT/SEC. STANDARD DEVIATION APPROX.	FRAG. VEL. 500FT/SEC MEAN FUZE ACTUATION DISTANCE	FRAG. VEL. 500FT/SEC STANDARD DEVIATION APPROX.
1.	160′	57′	80′	28′
2.	160′	170′	80′	85′



MEAN RADIUS OF BAR RING IN PLANE OF TARGET AREA = 60FT.
WIDTH OF RING ABOUT MEAN RADIUS = ±0FT.

CURVE No.	FRAG. VEL. 500 FT/SEC. MEAN FUZE ACTUATION DISTANCE	FRAG. VEL. 500 FT/SEC. STANDARD DEVIATION APPROX.	FRAG. VEL. 500 FT/SEC. MEAN FUZE ACTUATION DISTANCE	FRAG. VEL. 500 FT/SEC. STANDARD DEVIATION APPROX.
1.	240′	57′	80′	28′
2.	240′	340′	80′	170′



MEAN RADIUS OF BAR RING IN PLANE OF TARGET AREA. = 80FT.
WIDTH OF RING ABOUT MEAN RADIUS = ± 0FT.

CURVE No.	FRAG. VEL. 500 FT/SEC. MEAN FUZE ACTUATION DISTANCE	FRAG. VEL. 500 FT/SEC. STANDARD DEVIATION APPROX.	FRAG. VEL. 500 FT/SEC. MEAN FUZE ACTUATION DISTANCE.	FRAG. VEL. 500 FT/SEC. STANDARD DEVIATION APPROX.
1.	320′	57′	160′	28′
2	320′	340′	160′	170′

FIG. 6. THE VARIATION OF THE HIT CHANCE WITH R.M.S. MISS DISTANCE, WHEN THE FUZE ACTUATION DISTANCE IS ASSUMED TO BE DISTRIBUTED GAUSSIANLY ABOUT VARIOUS MEAN ACTUATION DISTANCES FOR VARIOUS STANDARD DEVIATIONS. THE WIDTH OF THE BAR RING IS TAKEN TO BE ± 0 FT. ABOUT THE MEAN RADIUS IN THE PLANE OF THE TARGET AREA.

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FIG. 7.

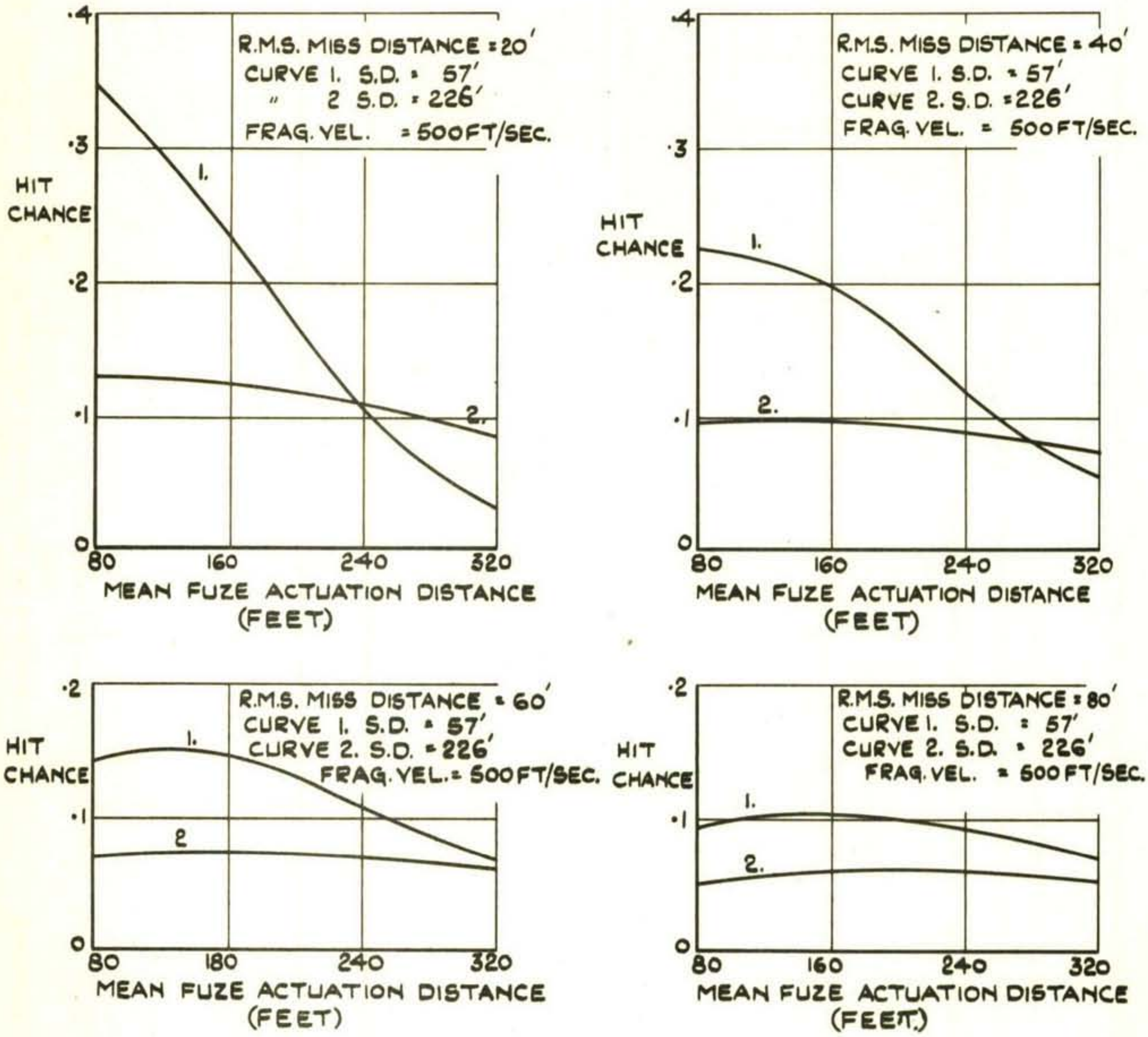
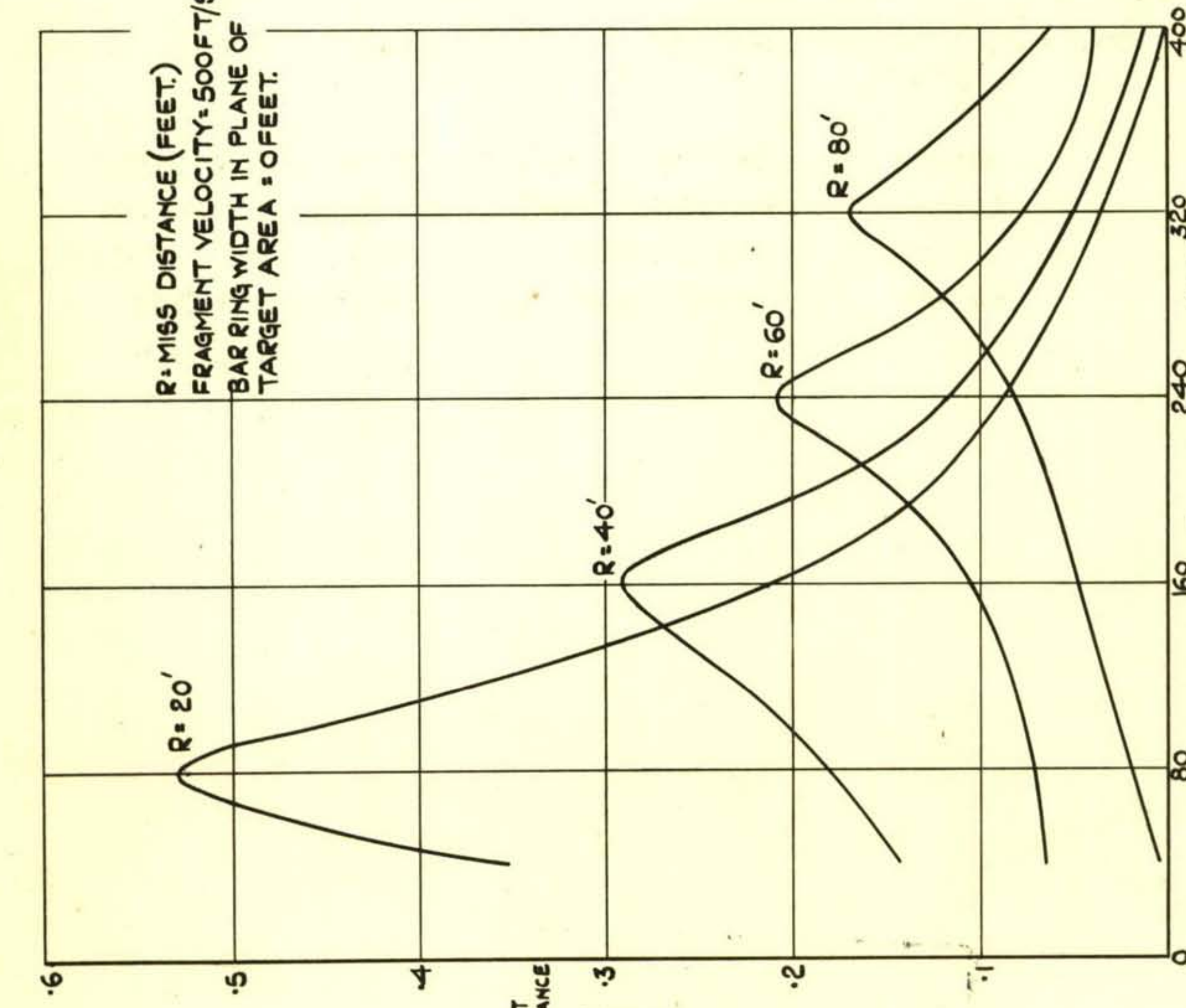
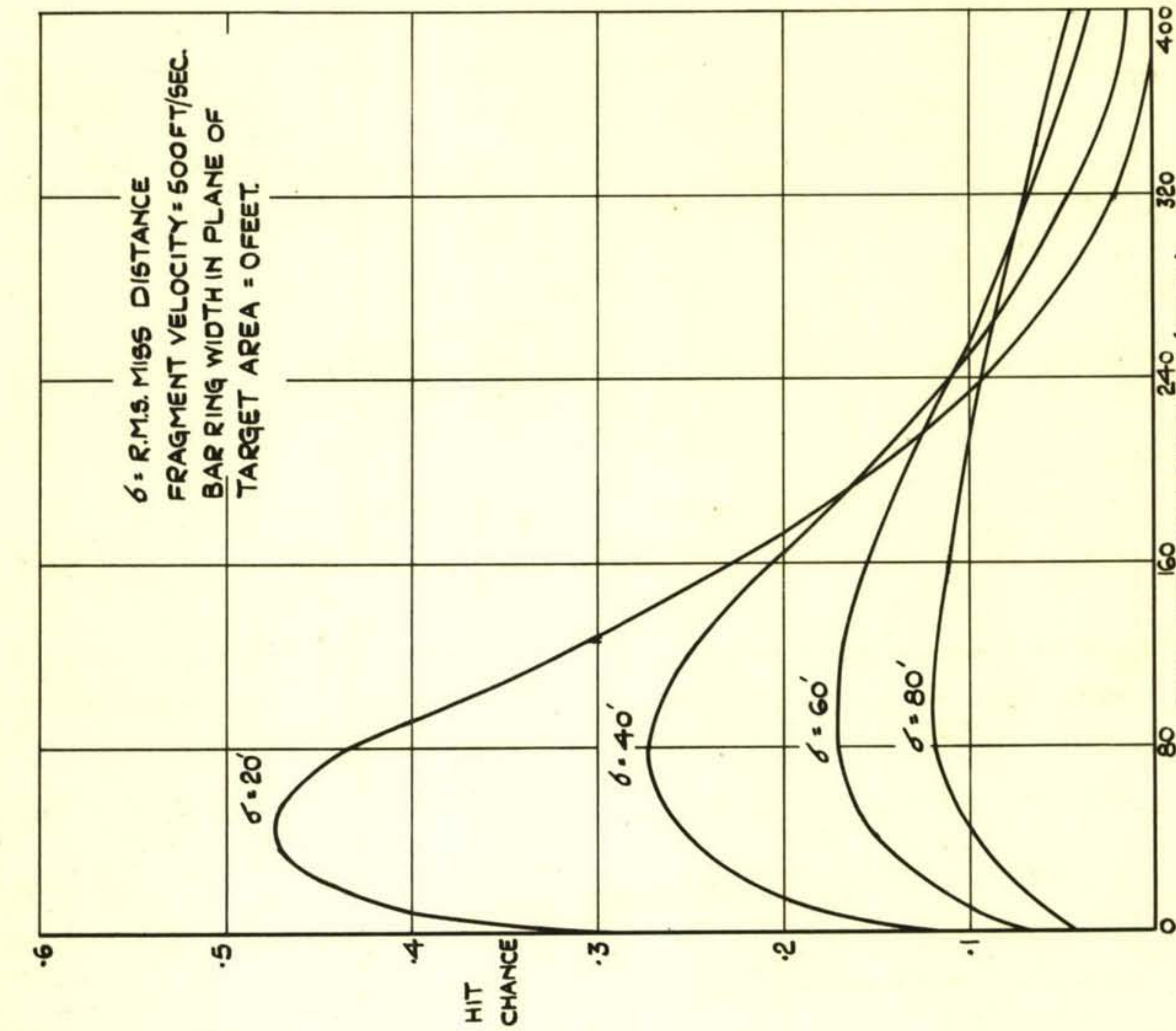


FIG. 7. VARIATION OF HIT CHANCE WITH MEAN FUZE ACTUATION DISTANCE WHEN THE FUZE ACTUATION DISTANCE IS GAUSSIANLY DISTRIBUTED ABOUT THESE MEANS WITH DIFFERENT STANDARD DEVIATIONS.

FIG.8. (a & b)



(a) THE VARIATION OF HIT CHANCE WITH THE FUZE ACTUATION DISTANCE FOR VARIOUS VALUES OF THE MISS DISTANCE.



(b) THE VARIATION OF HIT CHANCE WITH FUZE ACTUATION DISTANCE FOR VARIOUS R.M.S. VALUES OF THE MISS DISTANCE WHICH IS ASSUMED DISTRIBUTED GAUSSIANLY ABOUT ZERO MEAN.

FIG.8. a & b OPTIMUM FUZE ACTUATION DISTANCE.